

Oscillating and Inflating Universe in SO(3)-Gravitation Theory

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The predictions of SO(3) Gravitation Theory are studied for a homogeneous, isotropic universe. Various types of oscillations are obtained for the pre-inflationary phase, where the geometry experiences violent fluctuations. These are ultimately terminating at either re-collaps or regular inflation.

I. Introduction

Ten years after the invention of the first inflationary scenario [1], there is still a broad spectrum of opinions [2] concerning the extent to which the well-known difficulties of the standard cosmological model have been removed by that inflationary mechanism (for a review see [3–5]). Whereas some people are holding that nothing has been explained by inflation as long as the entropy problem for the primordial universe has not been clarified, other cosmologists take a moderate position by admitting that a least some of the problems might have been solved in a satisfactory way (e.g. monopole problem); finally there are the enthusiasts who see all the problems being well settled by inflation.

However, there seems to exist a sort of minimal consensus within the community of cosmologists; this refers to the general belief that despite the existing uncertainty about the detailed foundation of inflation (e.g. fine tuning problems) the very effect of an exponential growth of the universe yields such a highly plausible explanation of its past evolution that it must be true in one way or the other. Especially, those peculiar initial conditions of homogeneity and isotropy for the subsequent standard stage, as observed nowadays, become well understandable through the smoothing and flattening effects of inflation. Before the inflationary idea had been put forward as a purely classical effect, it was generally thought that those initial conditions could be produced exclusively by the late quantum era preceding immediately the classical standard phase.

In view of both the importance and the unclarified foundation of the inflationary mechanism, it may seem useful to reconsider the problem on a more fundamental level, i.e. by tracing it back to the question whether the original Einstein theory should be rigorously valid even on a cosmic scale. This eventually would imply that one does not have to bother about GUT phase transitions at the appropriate temperatures; rather one would prefer to look for some fundamental modification of the conventional gravitation theory such that its well-established validity is preserved on small scales (up to intergalactic distances, say); but its global cosmic predictions would have to be changed in order to embrace the inflationary effects as a naturally inherent feature, irrespective of the precise equation of state for the primordial matter.

A possible candidate for such a modification has recently been proposed [6–9] and the tentatively obtained solutions have clearly shown the occurrence of the desired inflationary effects. In the present paper, we further elaborate this point and especially study the transition from the pre-inflationary phase to the stage of exponential growth.

We proceed as follows: First, we condense the previous formulations of SO(3) gravitation theory into a brief summary, from which the two basic ideas of that approach may easily be read off: (i) the dispense with the Relativity Principle on a cosmic scale and (ii) the inclusion of the gravitational spin.

Next, we apply the equations of motion to the homogeneous and isotropic universe and study the occurrence of oscillations and inflation. It can be shown that inflation of the universe is the only stable configuration, so that all other solutions (e.g. oscillations or the standard solutions) ultimately terminate at such a de Sitter-like expansion phase. The only possibility for the universe, to escape this fate, is resorting

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to an early recollaps which however is attainable only for a closed model.

II. SO(3) Gravitation Theory

According to our hypothesis, the gravitational forces break the (local) Lorentz symmetry; i.e. the 4-world carries a spontaneous (1+3)-foliation and thus becomes a “space-time” manifold in the proper sense of the word. The dynamics of this foliation is what we call “gravitation”. Consequently, the gravitational variables will turn out as those geometric objects, which define that foliation. Clearly these are chosen as (i) the vector \mathbf{p} , which fixes the underlying 1-distribution $\check{\Delta}$, and (ii) the three vectors \mathcal{B}_i spanning the complementary 3-distribution $\hat{\Delta}$ (“characteristic distribution”), such that the tangent space T_x of space-time locally decomposes as

$$T_x = \check{\Delta}_x \oplus \hat{\Delta}_x. \quad (\text{II.1})$$

In terms of fibre bundles, the tangent bundle (τ_4) of space-time appears as the Whitney sum of the real line bundle $\check{\tau}_4$ and the 3-plane bundle $\hat{\tau}_4$:

$$\tau_4 = \check{\tau}_4 \oplus \hat{\tau}_4. \quad (\text{II.2})$$

Correspondingly, the associated principal bundle λ_4 with structure group $\text{GL}(4, \mathbb{R})$ reduces to the Whitney product of an $\text{GL}(1, \mathbb{R})$ bundle $\check{\lambda}_4$ and a $\text{GL}(3, \mathbb{R})$ bundle $\hat{\lambda}_4$: $\lambda_4 \rightarrow \bar{\lambda}_4 = \check{\lambda}_4 \times \hat{\lambda}_4$. The foliation fields $\{p^\mu, \mathcal{B}_i^\mu\}$ are pre-metric objects and are sometimes called the “ether fields”. In this pre-metric stage the various foliations may be distinguished only by the topologies of the integral manifolds of the corresponding distributions $\check{\Delta}, \hat{\Delta}$ (II.1). Therefore any two foliated “space-times” which are diffeomorphic to each other must be looked upon as being identical. Clearly, this circumstance enforces us to apply the Principle of General Covariance when looking now for the dynamics of foliation.

II.1. Metric

In general, any two adjacent integral manifolds will share the same topological properties. Therefore, the foliation can be taken as the proper dynamical object for a theory of gravitation only after the different integral manifolds have been equipped with some further structure, which may change continuously

across them and thus gives the handle for a dynamic law. Such an additional structure consists in a fibre metric in each of the two bundles $\check{\tau}_4, \hat{\tau}_4$ which then also yields a metric in the Whitney bundle τ_4 (II.2). The freedom for choosing an arbitrary (local) section $\mathcal{B}_\alpha^\mu(x) = \{p^\mu, \mathcal{B}_i^\mu\}(x)$ of the reduced bundle $\bar{\lambda}_4$ may be exploited in order to decompose the τ_4 metric \mathbf{G} with respect to the appropriately chosen ether fields

$$G_{\mu\nu} = \mathcal{B}_\mu^i \mathcal{B}_{i\nu} + p_\mu p_\nu \equiv g^{\alpha\beta} \mathcal{B}_{\alpha\mu} \mathcal{B}_{\beta\nu}. \quad (\text{II.3})$$

Here the appearance of the Minkowski metric $\mathbf{g} = \{\text{diag}(1, -1, -1, -1)\}$ indicates the fact that the ether fields are orthonormalized by the metric \mathbf{G} (II.3)

$$\mathcal{B}_{\alpha\mu} \mathcal{B}_\beta^\mu = g_{\alpha\beta}, \quad (\text{II.4})$$

so that the tetrad $\{\mathcal{B}_{\alpha\mu}\}$ may be considered as a local section of the principal Lorentz bundle \mathcal{A}_4 emerging here as the $\text{SO}(1, 3)$ reduction of λ_4 . Since that sub-bundle \mathcal{A}_4 further factorizes into the Whitney product of an $\text{SO}(1)$ and $\text{SO}(3)$ bundle, resp., i.e. $\mathcal{A}_4 \rightarrow \bar{\mathcal{A}}_4 = \mathcal{A}_4 \times \mathcal{A}_4$, according to the foliation hypothesis, there exists a residual $\text{SO}(3)$ degree of freedom which however is unobservable and therefore must be converted into a gauge artefact. Combining this result with the previously stated Principle of General Covariance leads to the implication that the desired foliation dynamics should appear in the shape of a coordinate-covariant $\text{SO}(3)$ gauge theory.

II.2. Connection

For the formulation of a dynamical law for the ether field, one first needs a connection (Γ) defining a coordinate-covariant derivative ∇ . A nearby choice is the Levi-Civita connection of the metric \mathbf{G} (II.3)

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} G^{\lambda\sigma} (\partial_\mu G_{\sigma\nu} + \partial_\nu G_{\sigma\mu} - \partial_\sigma G_{\mu\nu}), \quad (\text{II.5})$$

whose holonomy group is just the Lorentz group $\text{SO}(1, 3)$ by its very construction. Thus Special Relativity will keep up its *local* relevance, though our foliation hypothesis falsifies the Relativity Principle (at least for the gravitational interactions).

Let ω be the $\text{SO}(1, 3)$ gauge copy of Γ , then the ether fields identically satisfy the Cartan structure equations

$$\nabla_\mu \mathcal{B}_{\alpha\nu} = \mathcal{B}_{\beta\nu} \omega_{\alpha\mu}^\beta, \quad (\text{II.6})$$

which may also be re-written in terms of a combined $\text{SO}(1, 3) \times \text{GL}(4, \mathbb{R})$ covariant derivative $\hat{\nabla}$ as

$$\hat{\nabla}_\mu \mathcal{B}_{\alpha\nu} := \nabla_\mu \mathcal{B}_{\alpha\nu} + \omega_{\alpha\mu}^\beta \mathcal{B}_{\beta\nu} \equiv 0. \quad (\text{II.7})$$

However, since the boost degree of freedom of the Lorentz gauge group $SO(1, 3)$ must be frozen here according to the foliation hypothesis, we have to split the Lorentz connection ω into its rotational ($\bar{\omega}$) and boost ($\bar{\chi}$) parts:

$$\omega = \bar{\omega} - \bar{\chi}, \quad (\text{II.8 a})$$

$$\bar{\omega}_\mu = \bar{A}_{i\mu} L^i, \quad (\text{II.8 b})$$

$$\bar{\chi}_\mu = \mathcal{H}_{i\mu} l^i, \quad (\text{II.8 c})$$

where L^i (l^i) are the rotation (boost) generators. Thus the $SO(1) \times SO(3)$ covariant reformulation of the Lorentz gauge covariant identity (II.7) reads

$$\nabla_\mu p_\nu = \mathcal{H}_{i\mu} \mathcal{B}^i_\nu, \quad (\text{II.9 a})$$

$$\bar{\mathcal{D}}_\mu \mathcal{B}_{i\nu} = -\mathcal{H}_{i\mu} p_\nu, \quad (\text{II.9 b})$$

where the $SO(3)$ covariant derivative $\bar{\mathcal{D}}$ has been defined as usual:

$$\bar{\mathcal{D}}_\mu \mathcal{B}_{i\nu} \equiv \nabla_\mu \mathcal{B}_{i\nu} + \varepsilon_i^{jk} \bar{A}_{j\mu} \mathcal{B}_{k\nu}. \quad (\text{II.10})$$

Obviously the objects \bar{A} and \mathcal{H} fix the intrinsic and extrinsic geometry of 3-spaces of the foliation (to be denoted as the “characteristic surfaces”).

II.3. Foliation and Spin

The crucial point now is that we convert the meaning of (II.9 a)–(II.9 b) from an identity into a dynamical law for the ether fields (“ether dynamics”) by looking upon the $SO(3)$ connection $\bar{A}(x)$ (II.8 b) and the “Hubble vector” $\mathcal{H}(x)$ (II.8 c) as known space-time objects. Clearly, this procedure requires the specification of additional dynamical laws for those coefficient functions $\bar{A}(x)$, $\mathcal{H}(x)$ in order to obtain a closed dynamical system for the unique determination of the foliation (see below). However before facing this problem we want to draw attention upon the spin problem in General Relativity. It is generally held that the gravitational field should also carry spin (in the quantized theory the graviton appears as a spin-2 particle). But the traditional approach offers no handle to deal rigorously with the spin phenomenon (apart from the linearized theories). Concerning the present approach, the place where the spin could enter the theory is fairly self-suggesting; one merely has to change the connection $\bar{A}(x)$ (II.8 b) by some $SO(3)$ gauge vector \mathbf{C} (the “spin field”) into

$$A_{i\mu} = \bar{A}_{i\mu} - C_{i\mu}, \quad (\text{II.11})$$

which then has to be used in (II.10) in place of \bar{A} . The motivation for such an additional assumption is due to the expectation that the gravitational spin should play some role in a realistic gravitation theory. A possible answer to these questions is provided by the Cartan generalization of Riemannian geometry [10, 11]. Indeed it could easily be demonstrated [7] that the postulate (II.11) formally transcribes into the corresponding Cartan generalization of the Levi-Civita connection Γ (II.5). However, we believe that the physical space-time manifold is strictly Riemannian and consequently the spin effects must be adequately accounted for by an additional geometric object (our \mathbf{C} -field) in a *torsion-free* space-time. Though being always present, such an object is then neither part of the intrinsic Riemannian 4-geometry nor is it an ordinary matter field. But in any case, the number of the gravitational fields has been enlarged, and therefore we have to specify an additional dynamical law for the new spin field \mathbf{C} . This is accomplished in a natural way by looking now for the wave equations of the ether fields.

II.4. Wave Equations

It seems reasonable to assume that the gravitational interactions – as all other known forces – are subject to causal propagation. This implies that the spreading of local deformations of the foliation grid are governed by some wave mechanism. Since the foliation has been parametrized by the ether fields, the latter ones should turn out as particular solutions of some wave equation. This supposition is readily verified by differentiating once more the first order equations (II.9 a), (II.9 b) (however by use of \mathbf{A} (II.11) in place of \bar{A} (II.10)), which yields the following coupled Klein-Gordon equations:

$$\nabla^\mu \nabla_\mu p_\nu + \mathcal{M}^2 p_\nu = \eta_i \mathcal{B}^i_\nu, \quad (\text{II.12 a})$$

$$\mathcal{D}^\mu \mathcal{D}_\mu \mathcal{B}_{i\nu} + \mathcal{M}_{ij}^2 \mathcal{B}^j_\nu = -\eta_i p_\nu. \quad (\text{II.12 b})$$

Here, the mass parameters are found to be space-time dependent:

$$\mathcal{M}^2 = \mathcal{H}_{i\mu} \mathcal{H}^{i\mu}, \quad (\text{II.13 a})$$

$$\mathcal{M}_{ij}^2 = \mathcal{H}_{i\mu} \mathcal{H}_j^\mu + C_{i\mu} C_j^\mu - g_{ij} (C_{k\lambda} C^{k\lambda}) \quad (\text{II.13 b})$$

as well as the coupling function η

$$\eta_i = \mathcal{D}_\mu \mathcal{H}^{i\mu}. \quad (\text{II.14})$$

Moreover, the following two constraints have been used:

$$\varepsilon_i^{jk} \mathcal{H}_{j\mu} C_k^\mu = 0, \quad (\text{II.15 a})$$

$$\mathcal{D}_\mu C_i^\mu = 0, \quad (\text{II.15 b})$$

whose geometric meaning shall be elucidated below (“*symmetrization conditions*”). The first condition (II.15 a) establishes a weak correlation between the Hubble vector \mathcal{H} and the spin field \mathbf{C} , whereas the second one (II.15 b) is exclusively referred to the \mathbf{C} field. Therefore one can get rid of it by specifying the additional dynamical law (for the spin degree of freedom) in such a way that (II.15 b) is *automatically* satisfied.

For that purpose, one first defines the “*spin current*” \mathbf{j} through

$$j_{i\nu} = \frac{1}{4\pi c^2} \varepsilon_i^{jk} (\mathcal{D}_\nu \mathcal{B}_{j\mu}) \mathcal{B}_k^\mu, \quad (\text{II.16})$$

and then one observes that this current obeys the continuity equation

$$\mathcal{D}^\nu j_{i\nu} = 0, \quad (\text{II.17})$$

whenever the constraint (II.15 b) is satisfied. Indeed, one easily finds the relationship

$$j_{i\nu} = -\frac{1}{2\pi c^2} C_{i\nu}, \quad (\text{II.18})$$

where c is a constant length parameter. Consequently, we merely have to postulate a Yang-Mills type of equation as the desired additional dynamical law,

$$\mathcal{D}^\mu F_{i\mu\nu} = 4\pi j_{i\nu}, \quad (\text{II.19})$$

and the continuity equation (II.17) will be satisfied. The Yang-Mills field \mathbf{F} , emerging here, is due to the gauge potential \mathbf{A} (II.11) as usual,

$$F_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu} + \varepsilon_i^{jk} A_{j\mu} A_{k\nu}. \quad (\text{II.20})$$

Since the gauge potential \mathbf{A} as well as the current \mathbf{j} are strongly dependent upon the spin field \mathbf{C} , the Yang-Mills equation (II.19) may be considered as the dynamics for the spin, which then acts as its own source.

II.5. Foliation Energy

Once the wave equations have been established, it is very suggestive to equip the ether fields with that energy-momentum density which is provided by the canonical formalism. Indeed, imagining a fixed background geometry for the moment, it is easily verified

that the wave equations (II.12 a), (II.12 b), and (II.19) are the variational equations due to the Lagrangean \mathcal{L} ,

$$\mathcal{L} = \mathcal{L}_p + \mathcal{L}_{\mathcal{B}} + \mathcal{L}_F + \mathcal{L}_I, \quad (\text{II.21})$$

which is made up by four contributions, namely the ether part $(\mathcal{L}_p + \mathcal{L}_{\mathcal{B}})$:

$$\mathcal{L}_p = \frac{1}{8\pi c^2} [(\nabla_\mu p_\nu)(\nabla^\mu p^\nu) - \mathcal{M}^2 p^\mu p_\mu], \quad (\text{II.22 a})$$

$$\mathcal{L}_{\mathcal{B}} = -\frac{1}{8\pi c^2} [(\mathcal{D}_\mu \mathcal{B}_{i\nu})(\mathcal{D}^\mu \mathcal{B}^{i\nu}) - \mathcal{M}_{ij}^2 \mathcal{B}_\mu^i \mathcal{B}_\mu^j], \quad (\text{II.22 b})$$

the gauge field part \mathcal{L}_F :

$$\mathcal{L}_F = \frac{1}{16\pi} F_{i\mu\nu} F^{i\mu\nu}, \quad (\text{II.23})$$

and finally the interaction

$$\mathcal{L}_I = \frac{1}{4\pi c^2} \eta_i \mathcal{B}_\mu^i p^\mu. \quad (\text{II.24})$$

Correspondingly, the total energy-momentum density ${}^{(e)}\mathbf{T}$ (“*foliation energy*”) is composed of three parts:

$${}^{(e)}T_{\mu\nu} = {}^{(p)}T_{\mu\nu} + {}^{(\mathcal{B})}T_{\mu\nu} + {}^{(F)}T_{\mu\nu} \quad (\text{II.25})$$

because the interaction energy ${}^{(I)}\mathbf{T}$ due to \mathcal{L}_I (II.24) vanishes on account of the orthonormality constraint (II.4). Moreover it is easily verified that both Lagrangeans \mathcal{L}_p (II.22 a) and $\mathcal{L}_{\mathcal{B}}$ (II.22 b) are vanishing identically, when the foliation dynamics is inserted, which further simplifies the corresponding energy-momentum tensors ${}^{(p)}\mathbf{T}$ and ${}^{(\mathcal{B})}\mathbf{T}$.

The interesting point here is now that the foliation tensor ${}^{(e)}\mathbf{T}$ develops a non vanishing source in curved space,

$$\nabla_\mu {}^{(e)}T^\mu_\nu = {}^{(e)}m_\nu + {}^{(e)}s_\nu, \quad (\text{II.26})$$

with both the mass force \mathbf{m} and the spin force \mathbf{s} being due to the spin field \mathbf{C} :

$${}^{(e)}m_\lambda = \frac{1}{4\pi c^2} \partial_\lambda (C_{i\mu} C^{i\mu}), \quad (\text{II.27 a})$$

$${}^{(e)}s_\lambda = \frac{1}{2} R_{\sigma\mu\lambda\nu} {}^{(e)}\Sigma^{\sigma\mu\nu}. \quad (\text{II.27 b})$$

The spin density ${}^{(e)}\Sigma$ is given here in terms of the spin field \mathbf{C} through

$$\begin{aligned} {}^{(e)}\Sigma^{\sigma\mu\nu} &= \frac{1}{2\pi c^2} \varepsilon^{ijk} \mathcal{B}_i^\sigma C_j^\nu \mathcal{B}_k^\mu \\ &\equiv \varepsilon^{ijk} \mathcal{B}_i^\sigma \mathcal{B}_j^\mu j_k^\nu \end{aligned} \quad (\text{II.28})$$

and combines with the Riemannian curvature tensor \mathbf{R} to the Mathisson force \mathbf{s} (II.27 b) in the well-known way.

II.6. Einstein Equations

These results can now be used in order to write down the last dynamical equation, namely that for the coefficient functions $\bar{\mathbf{A}}, \mathcal{H}$ occurring in the foliation dynamics (II.9 a), (II.9 b), which then closes the whole dynamical system (apart from the matter equations). Clearly our choice consists in the well-known Einstein equations

$$E_{\mu\nu} = 8\pi L_p^2 T_{\mu\nu}, \quad (\text{II.29})$$

where we have used a rescaled energy-momentum tensor \mathbf{T} such that the Planck length L_p energies in place of Newton's gravitational constant. Observe here that the Riemannian \mathbf{R} is independent of the spin field: $\mathbf{R} = \mathbf{R}(\mathbf{p}, \mathcal{B}, \bar{\mathbf{A}}, \mathcal{H})$. Thus the spin field enters the Einstein equations (II.29) only on the right when putting

$$T_{\mu\nu} = {}^{(e)}T_{\mu\nu} + {}^{(m)}T_{\mu\nu}, \quad (\text{II.30})$$

where ${}^{(m)}\mathbf{T}$ is the energy-momentum tensor of ordinary matter. The goal here is that by this construction we are able to deal in *torsion-free* space-time also with *spinning* matter which in general develops Mathisson forces ${}^{(m)}\mathbf{s}$ in the presence of gravitational fields, i.e.

$$\nabla_\mu {}^{(m)}T^\mu_\nu = {}^{(m)}s_\nu \neq 0. \quad (\text{II.31})$$

This effect makes the traditional Einstein equations (II.29) inconsistent for $\mathbf{T} \equiv {}^{(m)}\mathbf{T}$. We have achieved the elimination of these inconsistencies of the Einstein theory by introducing our gravitational spin field \mathbf{C} just for the sake of the compensation of those unwanted forces,

$${}^{(m)}s_\mu + {}^{(e)}s_\mu + {}^{(e)}m_\mu = 0. \quad (\text{II.32})$$

In this context, we can now elucidate the geometric meaning of the previous constraints (II.15 a), (II.15 b). The crucial point for maintaining a torsion-free space-time namely is that the original Einstein equations (II.29) enforce a symmetric tensor \mathbf{T} (II.30) on their right-hand side! Since the matter tensor ${}^{(m)}\mathbf{T}$ should always be symmetric, this requirement implies the symmetry of the foliation tensor ${}^{(e)}\mathbf{T}$. However, it is a well known fact in flat-space field theory that spinning fields (like our ether fields) in general develop some asymmetry of their *canonical* energy-momen-

tum tensor ${}^{(C)}\mathbf{T}$ [10, 11]. But on the other hand, such an asymmetric tensor is *inconsistent* with the Einstein equations in curved space. To escape this dilemma one sometimes uses a new symmetric tensor \mathcal{T} in curved space (“*metric energy-momentum tensor*”) which differs from the canonical one ${}^{(C)}\mathbf{T}$ by certain spin terms:

$${}^{(C)}T_{\mu\nu} = \mathcal{T}_{\mu\nu} + \frac{1}{2} \nabla^\lambda (\Sigma_{\mu\lambda\nu} + \Sigma_{\nu\lambda\mu} + \Sigma_{\mu\nu\lambda}). \quad (\text{II.33})$$

In this way, the asymmetry of the canonical tensor ${}^{(C)}\mathbf{T}$ has been traced back to the spin effect. Conversely this fact can now be used to symmetrize the canonical tensor by imposing the “*symmetrization condition*”, namely

$$\nabla^\lambda \Sigma_{\mu\nu\lambda} = 0. \quad (\text{II.34})$$

In the particular case of our ether fields, where $\Sigma \rightarrow {}^{(e)}\Sigma$ (II.28) and ${}^{(C)}\mathbf{T} \rightarrow {}^{(e)}\mathbf{T}$ (II.25), the spin condition (II.34) is readily seen to be identical to the previous constraints (II.15 a), (II.15 b), if the foliation dynamics (II.9 a), (II.9 b) is taken into account. So we see that the present approach indeed is capable of consistently dealing with the spin phenomenon in a torsion-free space-time!

III. Oscillations and Inflation

As a consistency check of the theory developed so far, one may face a highly symmetric situation which is transparent enough to see the spin effect in action. Neglecting that effect ($\mathbf{C} \equiv 0$) should then lead back to the standard Einstein theory (\rightarrow “*standard solutions*”). Such a case, being well-suited for a comparison of both situations, is encountered in cosmology where the assumption of isotropy and homogeneity of the universe (*weak cosmological principle*) reduces the number of the relevant dynamical variables into one, namely the radius (r) of the universe. Consequently, the traditional Einstein equations yield an equation of motion for the radius r as a function of the cosmological time (t). The corresponding solutions depend upon the adopted equation of state for matter and then embrace what is called the “*standard cosmological model*”. As is well known this model is plagued with some serious difficulties, which however are generally held to be cured (or at least weakened) by the effects of inflation. Therefore it is an interesting question for looking to what extent the present modification of the standard Einstein theory may be capable of producing the desired inflation, which – when occurring – must

then be due to the spin effect. Especially we are interested in the transition from a standard solution to an inflationary configuration.

III.1. Equations of Motion

For a clarification of this question, we first write down the Einstein equations (II.29) for a homogeneous, isotropic universe, filled by matter of density m under pressure b , with the spin field \mathbf{C} being included in shape of a scalar function $s(t)$:

$$\ddot{r} = 2A^2 \frac{s^2}{r} - \left[\frac{(\sigma + s^2)^2}{r^3} + \frac{\dot{s}^2}{r} \right] - 4\pi r(b + \frac{1}{3}m), \quad (\text{III.1 a})$$

$$\dot{r}^2 = \sigma + \dot{s}^2 + \left(\frac{\sigma + s^2}{r} \right)^2 + \frac{8\pi}{3} r^2 m, \quad (\text{III.1 b})$$

$$\dot{m} = 3 \left[\frac{A^2}{2\pi} \frac{s^2}{r^2} - (m + b) \right] \frac{\dot{r}}{r} - \frac{3A^2}{2\pi} \frac{s\dot{s}}{r^2}. \quad (\text{III.1 c})$$

(All variables have been rescaled by the Planck length, see [9]). Whereas the first and third equations are the proper equations of motion, the second equation (III.1 b) serves as an “initial-value” equation correlating the initial values for the numerical integration. The length parameter A is the rescaled c of (II.18), i.e.

$$A = \frac{L_p}{c}. \quad (\text{III.2})$$

Further, the foliation index σ ($= \pm 1, 0$) denotes the open, closed and flat universes, resp. We are dealing in the following with the closed case exclusively ($\sigma = -1$). Moreover, we shall be satisfied with a linear approximation of the equation of state, i.e.

$$b = \beta m \quad (\beta = \text{const}). \quad (\text{III.3})$$

Here, $\beta = 0$ stands for the matter dominated and $\beta = \frac{1}{3}$ for the radiation-dominated universe.

The Einstein equations (III.1 a)–(III.1 c) must be complemented by the inhomogeneous Yang-Mills equation (II.19), which essentially is the equation of motion for the spin variable $s(t)$:

$$\ddot{s} = -\frac{\dot{r}}{r} \dot{s} + 2s \left(A^2 - \frac{\sigma + s^2}{r^2} \right). \quad (\text{III.4})$$

As expected, the whole dynamics (III.1 a)–(III.1 c) and (III.4) reduces for $s \equiv 0$ to the traditional Einstein equations exactly for $\sigma = 0$, and otherwise differs from the standard theory by the σ^2 -terms which are due to the foliation energy ${}^{(e)}T$ and therefore represent a sort of curvature energy of the characteristic surfaces.

III.2. Transient Solutions

The interesting point now is that the present dynamical equations do not only admit the known solutions of the standard cosmological theory ($s \equiv 0$) but also admit an empty de Sitter universe

$$s = \gamma_* r, \quad (\text{III.5 a})$$

$$r(t) = r_* \exp\left(\frac{t}{\tau}\right), \quad (\text{III.5 b})$$

$$m \equiv 0, \quad (\text{III.5 c})$$

$$\gamma_*^2 = \frac{A^2}{1 + A^2}, \quad (\text{III.5 d})$$

$$\frac{1}{\tau^2} = A^2 \gamma_*^2. \quad (\text{III.5 e})$$

This solution is exact in the case of a flat universe ($\sigma = 0$); and it is approximate for $\sigma = \pm 1$ in the regime where $s \gg |\sigma|$. Consequently, the question arises whether there is also a *transient* solution starting in the standard phase but gradually is transmuting into the inflationary phase (III.5 a)–(III.5 e)?

The result of the numerical integration plotted in Fig. 1 a gives a positive answer. As long as the initial value $s|_0 := s(t=0)$ is small enough, a standard re-collaps solution is slightly deformed but does not change its global character. However, if the excitations of the spin variable s are large enough, the re-collaps of the universe is suspended in favour of an inflationary phase whose initiation needs a short-lived stage of negative energy density m . After that intermediate stage, the (positive) mass density m is diluted infinitely by virtue of the rapid expansion (in agreement with known results [3]). The kind of transition to the de Sitter vacuum (III.5 a)–(III.5 e) may be conveniently read off from Fig. 1 b, which shows a plot of the spin variable s in terms of the radius r .

A further interesting effect is based upon the reflection invariance ($s \rightarrow -s$) of the equations of motion (III.1 a)–(III.1 c) and (III.4). This may be considered as a sort of “vacuum degeneration” (observe that the sign of γ_* for the de Sitter vacuum (III.5 a)–(III.5 e) is ambiguous). Consequently, one has to expect transient solutions of two types according to whether the final state is the positive ($\gamma_* > 0$) or negative ($\gamma_* < 0$) vacuum. Indeed the numerical integrations demonstrate the occurrence of both vacua for a radiation-dominated universe (Fig. 2 a, b); however the negative vacuum only occurs for smaller excitations of the spin field s . Clearly this must give rise to a new effect: since the

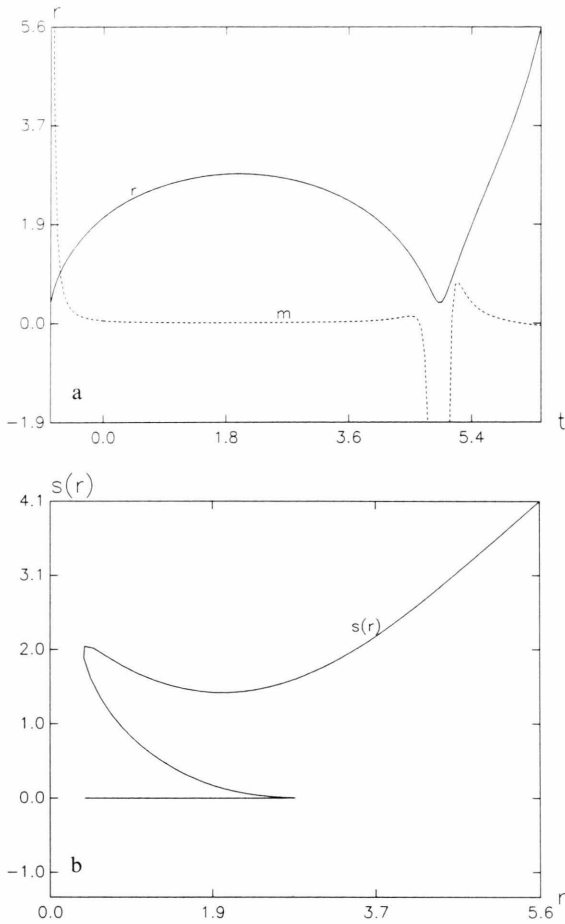


Fig. 1. Transient solutions for the closed ($\sigma = -1$), radiation-dominated ($\beta = \frac{1}{3}$) universe. For “larger” values of the spin variable s (here: $s|_0 = 4 \cdot 10^{-5}$), the final state becomes a positive de Sitter-vacuum ($\gamma_* > 0$). The spin variable s in terms of radius r approaches a straight line (III.5a) (Figure 1b). The mass density m (dotted) becomes negative in order to initiate inflation (Figure 1a). Initial-values: $r|_0 = 2$, $\dot{r}|_0 = 1$, $s|_0 = 0$; ($A = 1.5$).

choice of the final vacuum necessarily is discontinuous one must expect a qualitatively new behaviour for some intermediate value of the spin field!

III.3. Oscillations

Actually, Fig. 3 demonstrates the emergence of a double oscillation in that intermediate regime with a subsequent inflation phase of the positive type ($\gamma_* > 0$). The strong accelerations of the radius, for escaping the re-collaps, again require negative energy-densities for a short time. Further numerical studies have shown

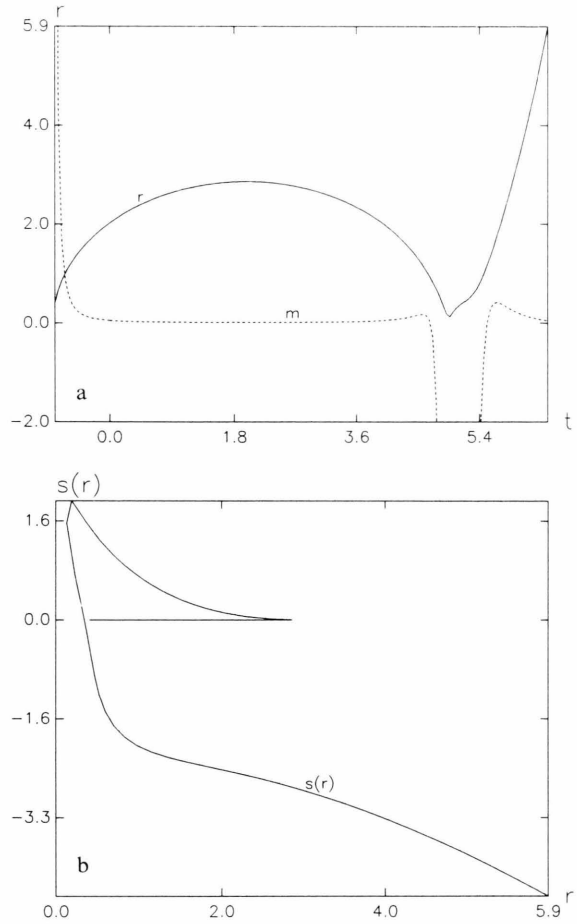


Fig. 2. de Sitter vacuum degeneration. For smaller excitations of the spin variable ($s|_0 = 3.67 \cdot 10^{-5}$), the final state is a *negative* de Sitter-vacuum ($\gamma_* < 0$). All other initial data and parameters are the same as for the *positive* vacuum (Figure 1).

that there also occurs a double oscillation with subsequent re-collaps, but we never did observe more than two oscillations nor a negative vacuum preceded by some oscillation (apart from the reflected solution $s \rightarrow -s$). It seems that the sign of the spin variable s during the oscillations also determines the type of the final vacuum state (Figure 3b).

The situation changes suddenly when one goes away a little bit from the perfect radiation dominance ($\beta = \frac{1}{3}$). Figure 4 shows a plot of what happens for $\beta = 0.32$ and $s|_0 \approx 3.07 \cdot 10^{-5}$ (all other initial data and parameters are unchanged). Obviously there are

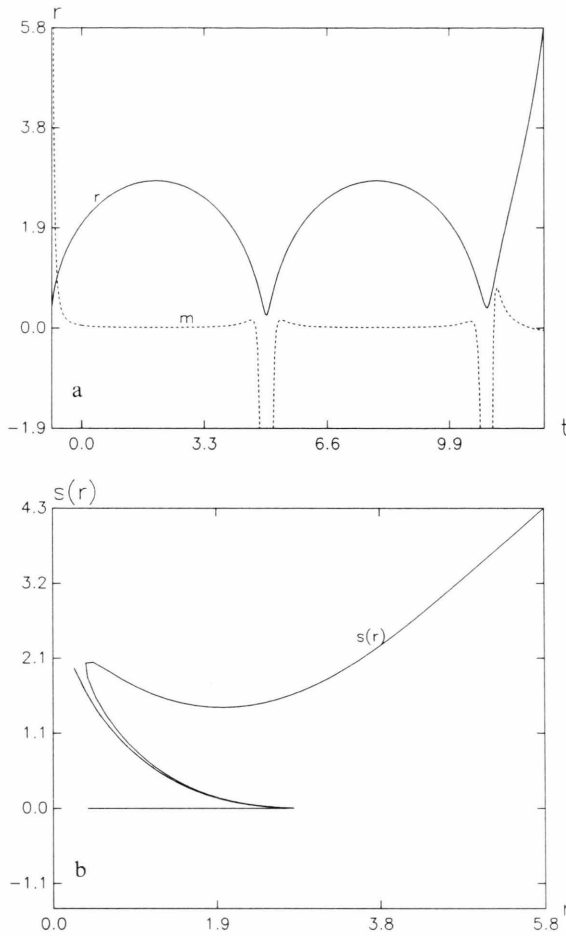


Fig. 3. Oscillations and inflation. A double oscillation with subsequent positive vacuum (Fig. 3a) or re-collapse occurs for intermediate values of the spin variable $s|_0 = 3.78 \cdot 10^{-5}$; all other parameters are unchanged relative to Figs. 1 and 2). The sign of s during the oscillation determines the type of vacuum (Figure 3b).

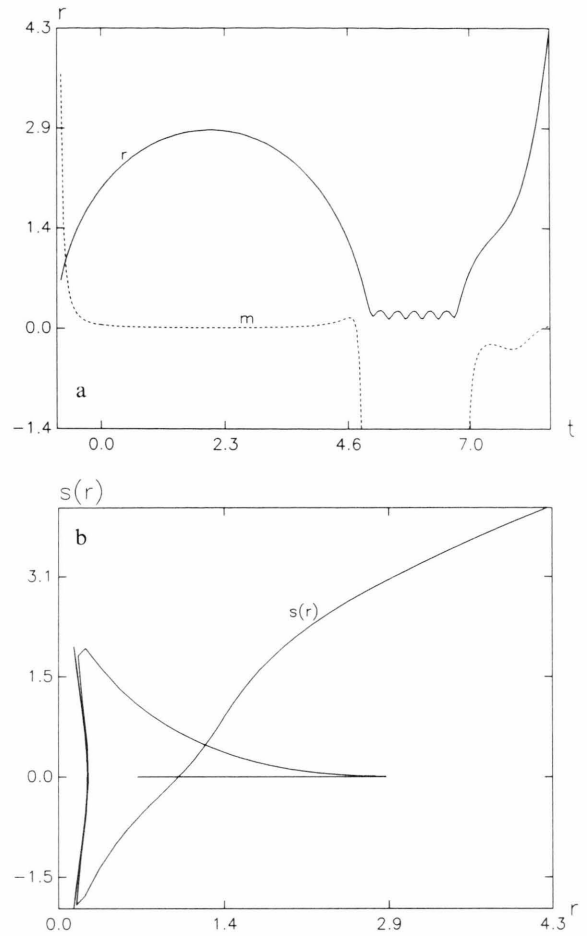


Fig. 4. Oscillations for radiation-nondominance ($\beta=0.32$). The first standard cycle is followed by small-amplitude oscillations (Figure 4a). The number of small oscillations can be regulated arbitrarily by means of the initial value $s|_0$ of the spin variable. The final state is again the positive de Sitter vacuum (Figure 4b).

oscillations after the first standard-like cycle, but these oscillations are of a quite different type (Fig. 4b): (i) the value of the radius r remains remarkably smaller than during the standard cycle, and (ii) the sign of s is changing concomitant with the phase of the oscillation. The final state is again the positive vacuum (Figure 4b). Contrary to the previous oscillations ($\beta=\frac{1}{3}$) the energy density m remains negative (at finite amount) during the whole oscillatory phase. It should be remarked that this type of behaviour roughly persists for all values of β down to the matter-dominance limit ($\beta=0$).

III.4. Discussion

The preceding results surely do not lack of a certain attractivity, because they roughly yield that picture which is to be expected from the pre-inflationary phase: namely a (more or less wildly) fluctuating geometry, from which the exponential expansion gradually is evolving. Furthermore, one may perhaps speculate also about a fluctuation of the topology of the characteristic surface; but since the topology as well as the high regularity of the 3-geometry (homogeneity and isotropy) have been put in by hand, the violent

fluctuations can only refer to the remaining dynamical variable r . However, one naturally tends to suppose that the most general solutions to the present theory will exhibit a fairly irregular behaviour of the geometry during the pre-inflationary stage of the evolution. The general belief is that this stage should be governed by quantum dynamics, but the subsequent inflation is held a purely classical effect. Therefore if the corresponding classical theory is extrapolated backwards into pre-inflation, it must yield a sort of classical analogue of the quantum fluctuations!

Though one can be satisfied in this respect with the present results of our purely classical theory, the situation becomes less favourable when one considers the post-inflationary stage. Adopting the general conviction that the universe nowadays is in a standard phase (according to the “standard model”), we must admit that the present theory predicts a rather unrealistic instability of such a standard state.

For a closer inspection of this assertion, one first considers the spin field s – in the vicinity of a standard solution ($s \approx 0$) – as a function of the radius r , which yields on account of the Yang-Mills equation (III.4)

$$\dot{r}^2 \frac{d^2 s}{dr^2} + \left(\ddot{r} + \frac{\dot{r}^2}{r} \right) \frac{ds}{dr} = 2s \left(\Lambda^2 - \frac{\sigma + s^2}{r^2} \right). \quad (\text{III.6})$$

Next we insert here the time derivatives of the radius r from the Einstein equation (III.1 a) and from the initial-value equation (III.1 b) with the mass density m for $s \equiv 0$ found from (III.1 c) as

$$\frac{8\pi}{3} r^2 m(r) \approx \left(\frac{r_{\max}}{r} \right)^{1+3\beta}. \quad (\text{III.7})$$

(Here, r_{\max} is the maximal radius, a horribly large number for the actual universe ($\sim 10^{60}$) when measured in Planck units). With these arrangements, the linearized Yang-Mills equation (III.6) reads for the radiation-dominated closed universe

$$(1 - \varrho^2) \frac{d^2 s}{d\varrho^2} - \varrho \frac{ds}{d\varrho} = 2s(1 + \lambda^2 \varrho^2) \quad (\lambda := \Lambda \cdot r_{\max}), \quad (\text{III.8})$$

where the radius r has been re-scaled to cosmic dimensions

$$\varrho := \frac{r}{r_{\max}}. \quad (\text{III.9})$$

Thus the spin variable $s(\varrho)$ is predicted by (III.8) to behave in the vicinity of maximal extension ($\varrho \lesssim 1$) approximately like

$$s(\varrho) \approx s(1) \cdot \exp(\pm 2 \sqrt{1 + \lambda^2} \cdot \sqrt{1 - \varrho}), \quad (\text{III.10})$$

which signals a horrible explosion even for moderate variations of the radius ϱ (observe $\lambda \approx 10^{60}$ for $\Lambda \approx 1$). As our numerical investigations have shown, such a rapid growth of s inevitably terminates in a de Sitter vacuum [12]. Moreover, similar results hold also for the other types of foliation ($\sigma = 0, 1$). Thus the exponential expansion of the de Sitter vacuum cannot be forced down to the moderate rate of the standard phase for a sufficiently long time.

But, probably it is a somewhat overdone claim if one would try to describe both the pre- and post-inflationary stages with one single equation of motion; at least one may doubt whether the equation of state (III.3) can be adopted as being exactly the same in both phases.

- [1] A. Guth, Phys. Rev. D **23**, 347 (1981).
- [2] A. Lightman and R. Brawer, Origins. The Lives and Worlds of Modern Cosmologists, Harvard University Press 1990.
- [3] S. K. Blau and A. H. Guth, in: 300 years of Gravitation (S. W. Hawking and W. Israel, eds.), Cambridge 1987. – A. Linde, *ibid.*
- [3] See the contribution of A. Linde in [3].
- [5] M. Turner, in: General Relativity and Gravitation, Proceedings of the 11th Conference on General Relativity and Gravitation (Stockholm 1986), Cambridge University Press 1987.
- [6] M. Mattes and M. Sorg, Nuovo Cim. B **106**, 273 (1991).
- [7] M. Mattes, Die Allgemeine Relativitätstheorie als SO(3)-Eichtheorie, doctoral thesis, Stuttgart 1990.
- [8] M. Mattes and M. Sorg, Non-Standard Model of the Expanding Universe, Preprint, Stuttgart 1991.
- [9] M. Mattes and M. Sorg, Non-Standard Approach to General Relativity, Preprint, Stuttgart 1991.
- [10] F. W. Hehl, Rep. Math. Phys. **9**, 55 (1976).
- [11] F. W. Hehl et al., Rev. Mod. Phys. **48**, 393 (1976).
- [12] This is the reason for choosing those small initial values for the spin variable s (see Fig. 1) in order to remain in the vicinity of a standard solution for a sufficiently long time.